# Experimental investigation of natural convection heat transfer in horizontal and inclined annular fluid layers 

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Received: 8 January 2007/Accepted: 28 August 2007/Published online: 11 September 2007
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#### Abstract

In the present study, an experimental investigation of heat transfer and fluid flow characteristics of buoyancy-driven flow in horizontal and inclined annuli bounded by concentric tubes has been carried out. The annulus inner surface is maintained at high temperature by applying heat flux to the inner tube while the annulus outer surface is maintained at low temperature by circulating cooling water at high mass flow rate around the outer tube. The experiments were carried out at a wide range of Rayleigh number ( $5 \times 10^{4}<R a<5 \times 10^{5}$ ) for different annulus gap widths ( $L / D_{\mathrm{o}}=0.23,0.3$, and 0.37 ) and different inclination of the annulus ( $\alpha=0^{\circ}, 30^{\circ}$ and $60^{\circ}$ ). The results showed that: (1) increasing the annulus gap width strongly increases the heat transfer rate, (2) the heat transfer rate slightly decreases with increasing the inclination of the annulus from the horizontal, and (3) increasing $R a$ increases the heat transfer rate for any $L / D_{\mathrm{o}}$ and at any inclination. Correlations of the heat transfer enhancement due to buoyancy driven flow in an annulus has been developed in terms of $R a, L / D_{\mathrm{o}}$ and $\alpha$. The prediction of the correlation has been compared with the present and previous data and fair agreement was found.


## List of symbols

A surface area of the inner tube
$D$ tube diameter
$F_{i j} \quad$ view factor
$G_{i} \quad$ irradiation

[^0]| Gr | Grashof number |
| :---: | :---: |
| $g$ | gravity acceleration |
| H | tube length |
| $\bar{h}$ | average heat transfer coefficient |
| I | electric current |
| $J$ | radiosity |
| $k$ | thermal conductivity of air |
| $k_{\text {w }}$ | thermal conductivity of end wall |
| $L$ | annulus thickness |
| $\overline{N u}$ | average Nusselt number |
| Pr | Prandtl number |
| $q$ | rate of heat transfer by convection from the inner tube |
| $q_{c}$ | rate of heat losses by conduction from the annulus end walls |
| $q_{r}$ | rate of heat transfer by radiation from the inner tube |
| $R a_{D_{0}}$ | Rayleigh number based on annulus outer diameter ( $D_{\mathrm{o}}$ ) |
| $R_{\text {o }}$ | annulus outer radius |
| $T$ | temperature |
| $T_{\mathrm{H}}$ | average surface temperature of the inner tube |
| $T_{\text {C }}$ | average surface temperature of outer tube |
| V | voltage |
| $\alpha$ | inclination angle measured from the horizontal |
| $\beta$ | coefficient of volume expansion |
| $\varepsilon$ | emissivity |
| $\sigma$ | Stefan-Boltzman constant |
| $v$ | kinematic viscosity |

## Subscripts

C cold
H hot
i inner
o outer
s side

## 1 Introduction

Natural convection heat transfer between two concentric tubes has received much attention because of its use in a wide variety of engineering applications such as inert gas insulation of high power electric cables, receivers of some focusing solar collectors and thermal energy storage systems. Kuhen and Goldstein [1, 2] carried out an experimental and numerical investigation of natural convection heat transfer for air and water in concentric and eccentric horizontal annuli for values of Rayleigh number up to $10^{7}$. Their work was carried out for a gap width to inner cylinder diameter ratio equal to 2.6 . In their experimental study, it was reported that the transition from laminar to turbulent flow occurs at Rayleigh number equal to $10^{6}$. Later, Bishop [3] and Mcleod and Bishop [4] reported experimental investigations of turbulent natural convection of helium between horizontal isothermal concentric cylinders of a radius ratio of 4.85 at cryogenic temperature. They reported that the heat transfer rate depends on the expansion number as well as the Rayleigh number. Also, several numerical investigations of turbulent natural convection in a concentric horizontal annulus have been conducted [5-10]. In these previous works, the influence of different effects and physical parameters as the annulus length, a finite rotation of the inner cylinder, multiple perturbation, transition from laminar to turbulence natural convection and very small radius difference were studied.

Comparing the results of the published papers in the field of natural convection in concentric annulus, it can be concluded that the rate of heat transfer and the critical Rayleigh number at which transition occurs to turbulent flow depends on the inner to outer diameter ratio. To the best of the author's knowledge, the specialized literature directly connected with experimental data to investigate the effects of the inner to outer diameter ratio and the annulus inclination angle on the fluid flow and heat transfer characteristics of annular fluid layer are very limited. Takata et al. [11] analytically investigated natural convection in an inclined cylindrical annulus enclosed in heated inner and cooled outer cylinder using the successive over-relaxation procedure. Later Hamad [12] and Hamad and Khan [13] reported that the annulus diameter ratio and the Rayleigh number influence on natural convection heat transfer in an annulus is more significant than the angle of inclination.

In this study a comprehensive experimental investigation of natural convection heat transfer in horizontal and inclined annuli is carried out for different annuli gap widths and annulus inclination angles. The study aims to investigate the effects of the inclination angle, annulus gap widths and Rayleigh number on the fluid flow and
heat transfer characteristics. Also the study aims to deduce a general correlation that can predict the rate of heat transfer by natural convection in concentric annuli in terms of the Rayleigh number, inclination angle and annulus gap width. This correlation will be useful and can be used as a design guide line in a wide range of engineering applications.

## 2 Experimental setup and procedure

### 2.1 Experimental setup

The experimental set up consists of a concentric group of electric heater rod and two copper tubes of diameters 37 and 75 mm , respectively. The length of this group is 400 mm . This group forms two annuli as shown in Fig. 1. The first annulus is bounded by the heating rod surface (the diameter of this rod is variable) and the inside surface of the $37-\mathrm{mm}$ diameter tube. The ends wall of this annulus is made of $10-\mathrm{mm}$ thick Plexiglas sheet of very low thermal conductivity to reduce heat losses through the end walls. The trapped air in this annulus transfer heat by natural convection from the heater hot surface to the cold surface of the $37-\mathrm{mm}$ diameter tube. The second annulus is bounded by the outside surface of the $37-\mathrm{mm}$ diameter tube and the inside surface of the $75-\mathrm{mm}$ diameter. The ends side walls of this annulus is made of $2-\mathrm{mm}$ thick copper sheet. A cooling water is circulated in this annulus at high mass flow rate to maintain the surface of the inside tube isothermally at cold temperature. This concentric group was mounted on a rotary frame to vary the annulus inclination angle $(\alpha)$ from $0^{\circ}$ to $60^{\circ}$; measured from the horizontal.

The heating rod consisted of nickel-chrome wire wound round a Ceramic rod and insulated with a mica shell. The Ceramic rod was centrally inserted inside a thin polished copper tube that represents the inner surface of the air annulus. The gab between the Ceramic rod and this copper tube was filled by very fine sand. The copper tube was Teflon plugged from both ends. Three different electric heaters of different outside diameter $(12,18,25 \mathrm{~mm})$ of the outer copper tube were used in this work to obtain different annulus gap widths. A cross section view of the annuli and the heater rod is shown in Fig. 1. The heater was connected with a DC power supply to control the power input. The voltage and current supplied to the heater were measured by digital voltmeter and ammeter of accuracy $0.025 \%$. The temperature distribution of the inner surface of the air annulus was measured using 20 Teflon coated thermocouples (type J) distributed on five equally spaced axial location. Each axial location contains four thermocouples

Fig. 1 Experimental set up


Section of Heating Rod
equally distributed around the circumference of the heater outer surface as shown in Fig. 1. To facilitate the installation of thermocouples without disturb the free convection currents inside the air annulus, holes were drilled in the copper tube that cover the heating rod. The thermocouples were inserted from these holes and their junctions were fixed on the outer surface of the tube and the thermocouples wires were passed in the sand gab of the heating rod (see Fig. 1). To estimate conduction heat losses across the annulus ends walls, two thermocouples (type J) were fixed on the inner and outer surfaces of each end wall of the inner annulus. The temperature of the outer surface of the air annulus was measured by two thermocouples fixed on this surface near the locations of the inlet and exit of the circulating water. The water was circulated at mass flow rate of $0.03 \mathrm{~kg} / \mathrm{s}$. At this mass flow rate, the maximum temperature rise of the water between the exit and the inlet that occurred at the highest value of Rayleigh number $\left(5 \times 10^{5}\right)$ in the experimental program was $0.15^{\circ} \mathrm{C}$. This mass flow rate of the cooling water with $0.15^{\circ} \mathrm{C}$ variation in its temperature in addition to the high thermal conductivity (copper) of the annulus outer tube are able to maintain a uniform temperature on the inside surface of the annulus outer tube. The measurements reveals that the difference between the readings of the thermocouples fixed on the outer surface of the air annulus was in the range $0.1-0.4^{\circ} \mathrm{C}$ according to the Rayleigh number (power input). All thermocouples were calibrated in a constant temperature path using standard thermometer of $\pm 0.1^{\circ} \mathrm{C}$. All the temperature signals were acquired with a data acquisition system and sent into a PC for data recording.

### 2.2 Experimental condition

The ranges of the tested variables used in this study were:
Dimensionless annulus gap widths $0.23,0.3$ and 0.37 $\left(L / D_{\mathrm{o}}\right)$
Inclination angles
$0^{\circ}, 30^{\circ}, 60^{\circ}$
Rayleigh number based on $D_{o}$

$$
5 \times 10^{4}-5 \times 10^{5}
$$

### 2.3 Experimental procedure

The procedure and the experimental program were as follows:

1. Mount the electric heater of the appropriate diameter to obtain a certain annulus gap width.
2. Adjust the annulus inclination angle to the required value.
3. Allow the cooling water to circulate around the annulus outer tube at high flow rate.
4. Adjust the input power to the heater to obtain a certain Rayleigh number.
5. Allow the experiment to run for a long until steady state condition was achieved. The steady state condition was considered to be achieved when the differences in the measured temperatures were not more than $0.2^{\circ} \mathrm{C}$ over 30 min . In all the experiments, the steady state condition period was within 3-4 h according to the parameters of the experiment ( $\mathrm{Ra}, L /$ $D_{\mathrm{o}}$ and $\alpha$ ) and the number of the experiment in the day.
6. After steady state condition has been established, the readings of all thermocouples, the input power and the ambient temperature were recorded.
7. Repeat steps $4-5$ for five different Rayleigh numbers varying from $5 \times 10^{4}$ to $5 \times 10^{5}$
8. Repeat steps $2-7$ three times for different inclination angles; $0^{\circ}, 30^{\circ}$ and $60^{\circ}$.
9. Repeat steps $1-8$ three times for different $L / D_{\mathrm{o}} ; 0.23$, 0.3 and 0.37 .

## 3 Data reduction

To enable the separately studying of the effects of the Rayleigh number, the annulus gap width and the annulus inclination angle on the heat transfer rate, the Rayleigh number in this study will be defined based on the annulus outer diameter, (the annulus outer diameter is the dimension that was fixed in all the experiments). The Rayleigh number was calculated from the measured quantities using the definition:
$R a_{D_{\mathrm{o}}}=\frac{\rho C_{p} g \beta\left(T_{\mathrm{H}}-T_{\mathrm{C}}\right) D_{\mathrm{o}}^{3}}{k v}$
Where $D_{\mathrm{o}}$ is the annulus outer diameter, $g$ is the acceleration of gravity and $\rho, c_{p} \beta, k, v$ are density, specific heat at constant pressure, coefficient of volume expansion, thermal conductivity, and kinematic viscosity of air, respectively. All these properties were taken at $\left(T_{\mathrm{H}}+T_{\mathrm{C}}\right) / 2$, where $T_{\mathrm{H}}$ is the average temperature of the annulus inner surface (Taken as the average of the readings of the 20 thermocouples mounted on this surface) and $T_{\mathrm{C}}$ is the average temperature of the annulus cold outer surface (taken as the average of the readings of the two thermocouples fixed on this surface). Temperature measurements showed that the variation of the surface temperature of the annulus inner surface was within $2^{\circ} \mathrm{C}$, and the deviation between the readings of the two thermocouples mounted on the enclosure cold surface was within $0.4^{\circ} \mathrm{C}$. The energy balance for the annulus gives
$V I=q+q_{c}+q_{r}$
where $I$ and $V$ are electric current and voltage input to the heater, $q$ is the heat transfer by free convection from annulus inner hot surface to the annulus outer cold surface through the air layer enclosed in the annulus, $q_{c}$ is the heat losses by conduction through the annulus ends walls and $q_{r}$ is the heat transfer by radiation from the annulus inner hot surface and the annulus ends walls to the annulus outer cold surface. To minimize the conduction heat losses through these end walls, the material (plexiglas of low thermal conductivity) and thickness ( 10 mm ) were chosen. Moreover, two thermocouples are fixed on the opposite
sides of each wall to approximately estimate this conduction heat losses. The conduction heat loss from the annulus end walls was calculated as follows
$q_{c}=\frac{A_{\mathrm{s}}\left(\Delta T_{\mathrm{s} 1}+\Delta T_{\mathrm{s} 2}\right)}{\left(t_{\mathrm{s}} / k_{\mathrm{s}}\right)}$
where $A_{\mathrm{s}}$ is the area of the side wall, $k_{\mathrm{s}}$ is the thermal conductivity of the plexiglas glass side walls, $t_{\mathrm{s}}$ is the thickness of the Plexiglas end walls, and $\Delta T_{\mathrm{s} 1}$ and $\Delta T_{\mathrm{s} 2}$ are the temperature differences across the side walls, respectively. The conduction heat losses through the annulus end walls was within $1 \%$ of the input power in all the experiments.

The radiation was incorporated in the losses based on the radiosity/irradiation formulation. The four interior surfaces of the annulus are assumed to be opaque, diffuse, isothermal and gray. The radiation heat loss $q_{r}$ from the annulus is the net rate at which radiation incident on the outer cold surface of the annulus $q_{r, \mathrm{o}}$ and is calculated from
$q_{r}=q_{r, \mathrm{o}}=A_{\mathrm{o}} F_{\mathrm{oi}}\left(J_{\mathrm{o}}-J_{\mathrm{i}}\right)+A_{\mathrm{o}} F_{\mathrm{os}}\left(J_{\mathrm{o}}-J_{\mathrm{s}}\right)$
where $F_{\text {oi }}$ and $F_{\text {os }}$ are the view factor between the outer surface and the inner and side surfaces, respectively and $J_{\mathrm{i}}$, $J_{\mathrm{o}}$ and $J_{\mathrm{s}}$ are the radiosity of the inner, outer and side surfaces of the annulus and are given by
$J_{\mathrm{i}}=\varepsilon_{\mathrm{i}} \sigma T_{\mathrm{H}}^{4}+\left(1-\varepsilon_{\mathrm{i}}\right)\left(F_{\mathrm{io}} J_{\mathrm{o}}+F_{\mathrm{is}} J_{\mathrm{s}}\right)$
$J_{\mathrm{o}}=\varepsilon_{\mathrm{i}} \sigma T_{\mathrm{C}}^{4}+\left(1-\varepsilon_{\mathrm{o}}\right)\left(F_{\mathrm{oi}} J_{\mathrm{i}}+F_{\mathrm{os}} J_{\mathrm{s}}\right)$
$J_{s}=\varepsilon_{\mathrm{s}} \sigma T_{\mathrm{s}}^{4}+\left(1-\varepsilon_{\mathrm{s}}\right)\left(F_{\mathrm{so}} J_{\mathrm{o}}+F_{\mathrm{si}} J_{\mathrm{i}}\right)$
The view factors $F_{\mathrm{io}}, F_{\mathrm{is}}, F_{\mathrm{oi}}, F_{\mathrm{os}}, F_{\mathrm{si}}$ and $F_{\text {so }}$ were calculated based on the graphs and expressions given in Suryanarayana [15]. Equations 4-7 were solved together to find $q_{r}$ in terms of the surfaces temperatures of the annulus walls. The radiation heat losses were within $5 \%$ of the input power, respectively.

The average heat transfer coefficient based on the annulus outer surface can be found from
$\bar{h}=\frac{q}{\pi D_{\mathrm{o}} H\left(T_{\mathrm{H}}-T_{\mathrm{C}}\right)}$
The Nusselt number based on this average heat transfer coefficient and the annulus outer diameter can be found from
$\overline{N u}=\frac{\bar{h} D_{\mathrm{o}}}{k}=\frac{q}{\pi H k\left(T_{\mathrm{H}}-T_{\mathrm{C}}\right)}$
In annulus free convection, the heat transfer rate depends on $\ln \left(D_{\mathrm{o}} / D_{\mathrm{i}}\right)$, which is not included in the definition of $\bar{h}$ and $\overline{N u}$, and therefore the heat transfer rate is
commonly expressed in a form that does not involve the convection heat transfer coefficient as in the usual sense [1, $2,14,15,17]$. It is expressed in terms of the effective thermal conductivity ( $k_{\text {eff }}$ ) which is defined as the thermal conductivity that a stationary fluid should have to transfer the same amount of heat as the buoyancy driven moving flow. Therefore, the heat transfer rate by free convection in an annulus can be expressed as:
$q=\frac{2 \pi H k_{\text {eff }}}{\ln \left(D_{\mathrm{o}} / D_{\mathrm{i}}\right)}\left(T_{\mathrm{H}}-T_{\mathrm{C}}\right)$
Where $D_{\mathrm{i}}$ and $H$ are the annulus inner diameter and the annulus length, respectively. Combining Eqs. 9 and 10, the ratio of the effective thermal conductivity to the thermal conductivity of the air fluid layer in case of pure conduction can be expressed as follows:
$\frac{k_{\text {eff }}}{k}=\overline{N u} \operatorname{Ln}\left(D_{\mathrm{o}} / D_{\mathrm{i}}\right)=\frac{q \operatorname{Ln}\left(D_{\mathrm{o}} / D_{\mathrm{i}}\right)}{2 \pi H k\left(T_{\mathrm{H}}-T_{\mathrm{C}}\right)}$
In the present work the heat transfer enhancement due to the buoyancy effect are expressed in terms of $\left(k_{\text {eff }} / k\right)$ as it has been done by most of previous work and heat transfer text books [1, 2, 14, 15, 17].

Combining Eqs. (2-11) together, the expression of $\left(k_{\text {eff }} / k\right)$ can put as function of all the variables that affect the experimental determination of it. That is
$k_{\text {eff }} / k=f\left(T_{\mathrm{H}}, T_{\mathrm{C}}, T_{\mathrm{s}}, D_{\mathrm{o}}, D_{\mathrm{i}}, \varepsilon, k_{\mathrm{s}}, \ldots\right)$
The uncertainty $\Delta\left(k_{\text {eff }} / k\right)$ in the value of $\left(k_{\text {eff }} / k\right)$ was estimated based on the procedure of Holman and Gajda [16] and is expressed as follows
$\Delta\left(k_{\mathrm{eff}} / k\right)=\sqrt{\sum_{i=1}^{n}\left(\frac{\partial\left(k_{\mathrm{eff}} / k\right)}{\partial x_{i}} \Delta x_{i}\right)^{2}}$
where $\Delta x_{i}$ is the uncertainty in the $x_{i}$ variable that effect the experimental determination of $\left(k_{\text {eff }} / k\right)$. The uncertainty in the various variables used in the determination of the ( $k_{\text {eff }} / k$ ) were: $0.25 \%$ for the electric current $I, 0.25 \%$ for the electric volt $V, 0.2^{\circ} \mathrm{C}$ for any temperature measurement, 0.001 m for any distance measurement, $0.5 \%$ for the thermal conductivity of air, $2 \%$ for the thermal conductivity of plexiglass, and $5 \%$ for the emittance of the annulus surfaces. It was found that the uncertainty for the data of ( $k_{\text {eff }} / k$ ) ranges from 5 to $8 \%$.

## 4 Results and discussions

The experimental work was performed to study the effects of annular gap width $\left(L / D_{\mathrm{o}}\right)$, and annulus inclination angle
$(\alpha)$ on the heat transfer coefficient by free convection between the hot annulus inner surface and the cold outer annulus surface for a wide range of Rayleigh numbers. For each annulus gap width and annulus inclination, ( $k_{\text {eff }} / k$ ) was obtained for different Rayleigh numbers varying from $5 \times 10^{4}$ to $5 \times 10^{5}$. The effective thermal conductivity was found to be strongly dependent on the annulus gap width and Rayleigh number and slightly dependent on annulus inclination angle. Figures 2, 3, 4, 5, 6 and 7 include the variation of ( $k_{\text {eff }} / k$ ) with $R a$ for different annulus widths and at various annulus inclination angles.

### 4.1 Effect of annulus gap width

Figures 2, 3 and 4 show the effect of annulus gap $\left(L / D_{\mathrm{o}}\right)$ on the enhancement rate of heat transfer, $\left(k_{\text {eff }} / k\right)$, at the entire range of Rayleigh numbers for annulus inclination angles of $0^{\circ}, 30^{\circ}$ and $60^{\circ}$, respectively. As shown in the figures, for any annulus inclination angle and at the entire range of Rayleigh number ( $k_{\text {eff }} / k$ ) increases with increasing annulus gap width $\left(L / D_{o}\right)$. Buoyancy-driven flow in a horizontal annulus bounded by concentric tubes is characterized by two convection cells that are symmetric about the vertical midplane. In the case of heating the annulus inner surface and cooling the annulus outer surface, as in the present study, fluid ascends and descends along the inner and outer annulus surfaces, respectively. When the annulus gap width decreases, the resistance to the circulation motion of the convection cells increases and this leads to slower replacement of the hot air adjacent to the inner annulus surface by the cold air adjacent to the annulus outer surface and this results in an increase of the average temperature of the annulus inner surface and consequently a decrease of the heat transfer rate. Also Figs. 2, 3 and 4 show that the curves of the different $L / D_{\mathrm{o}}$ ratios converges to each others as $R a$ decreases. This means that the effect of the gap width


Fig. 2 Effect of annulus gap width on heat transfer rate for $\alpha=0^{\circ}$


Fig. 3 Effect of annulus gap width on heat transfer rate for $\alpha=30^{\circ}$


Fig. 4 Effect of annulus gap width on heat transfer rate for $\alpha=60^{\circ}$


Fig. 5 Effect of annulus inclination angle on heat transfer rate for $L / D_{\mathrm{o}}=0.24$
on the heat transfer rate decreases with decreasing $R a$. This can be attributed to that as $R a$ decreases the heat convection becomes insignificant, or in other words, heat conduction becomes the dominant heat transfer in the fluid layer. This observation is consistent with the results of


Fig. 6 Effect of annulus inclination angle on heat transfer rate for $L / D_{\mathrm{o}}=0.3$


Fig. 7 Effect of annulus inclination angle on heat transfer rate for $L / D_{\mathrm{o}}=0.37$

Kuehn and Goldstien [1, 2]. They have shown that the heat convection in concentric cylinders becomes the dominant heat transfer mode as $R a_{L}$ increased over 1,000 (critical value), where $R a_{L}$ is defined based on the gap width, $L$.

### 4.2 Effect of annulus inclination angle

Figures 5, 6 and 7 show the effect of the annulus inclination angle on the enhancement in heat transfer rate, $\left(k_{\text {eff }} / k\right)$, for the three annulus gap widths and at the entire range of $R a$. As shown in the figures, for any annulus gap width ( $k_{\text {eff }} / k$ ) slightly decreases with increasing the annulus inclination from the horizontal. This may be attributed to the increase of the convection current path of the convection cells with increasing its inclination from the horizontal. Increasing this length decreases the rate of replacement of the hot air adjacent to the annulus inner surface by the cold air adjacent to the annulus outer surface and this decrease the heat transfer rate. Comparing Figs. 5,

6 and 7 one can notice that the effect of the inclination angle on the heat transfer rate decreases as the annulus gap width decreases. This can be attributed to that as the gap width becomes thinner the increase of the convection current path with the inclination angle decreases.

### 4.3 Effect of Rayleigh number

Figures 2, 3, 4, 5, 6 and 7 show the variation of $\left(k_{\text {eff }} / k\right)$ with $R a$ for different annulus gap widths and at different annulus inclination. The figures show that for any annulus gap width and annulus inclination angle, ( $k_{\text {eff }} / k$ ) generally increases with increasing $R a$. This can be attributed to the increase of the buoyancy force with increasing Rayleigh number; i.e., with increasing the temperature gradient or ( $T_{\mathrm{H}}-T_{\mathrm{C}}$ ). Increasing the buoyancy force increases the flow driving force and in consequently causes an increase of flow intensity that leads to higher heat transfer rates. Also increasing Rayleigh number enhances the mixing within the fluid due to the increase of the turbulence of the vortices and these leads to better heat transfer performance.

## 5 Empirical correlations

A general empirical correlation was deduced from the present experimental data to give ( $k_{\mathrm{eff}} / k$ ) in terms of Rayleigh number, annulus gap width and annulus inclination angles. The characteristics and physics of heat transfer data, shown in Figs. 2, 3, 4, 5, 6 and 7 reveal that: (1) the heat transfer rate strongly increases with increasing $R a$ and $L / D_{\mathrm{o}}$ and the variation takes the form of a power function and slightly decreases with increasing the annulus inclination from the horizontal and the variation takes the form of a polynomial functions. Therefore the experimental data are fitted to give ( $k_{\text {eff }} / k$ ) as a power functions of $R a$ and $L /$ $D_{\mathrm{o}}$ and as a second-order polynomial function of $\cos \alpha$. The expression for this correlation is developed in the following form to have a minimum error:
$K_{\text {eff }} / K=1.21 R a^{0.33}\left(L / D_{\mathrm{o}}\right)^{0.99}(1-0.34 \cos \alpha(1-1.06 \cos \alpha))$

This correlation was developed under the following studied ranges: $5 \times 10^{4} \leq R a \leq 5 \times 10^{5}, 0.23 \leq L / D_{\mathrm{o}} \leq 0.37$ and $0^{\circ} \leq$ $\alpha \leq 60^{\circ}$.

Figures 2, 3, 4, 5, 6 and 7 depict comparison between the correlation prediction and the experimental data. The agreement between the correlation prediction and the corresponding experimental data is within the experimental


Fig. 8 Deviation of experimental data from correlation's prediction
uncertainty. Also, Fig. 8 shows that Eq. 14 can predict all the experimental data within $\pm 8 \%$

## 6 Comparison with literature

To the best of our knowledge there is no previous work was found in the literature that studied the effect of the inclination angle on heat transfer and fluid flow characteristics of buoyancy driven flow in inclined annulus. Therefore, comparison of the present work of inclined annulus with that in the literature can not be achieved. However, the present work of horizontal annulus can be compared with previous works. Figure 9 compares ( $k_{\text {eff }} / k$ ) of the present study for horizontal annulus with the prediction of Raithby and Hollands [17] correlation's. Raithby and Hollands [17] correlated their data in terms of $R a_{c}^{*}$, where


Fig. 9 Comparison between present data and previous experimental data
$R a_{c}^{*}=\frac{g \beta\left[\ln \left(D_{\mathrm{o}} / D_{\mathrm{i}}\right)\right]^{4}\left(T_{\mathrm{H}}-T_{\mathrm{C}}\right)}{\left(D_{\mathrm{i}}^{-3 / 5}+D_{\mathrm{o}}^{-3 / 5}\right)^{5} \alpha \nu}$
Therefore, to compare the present work with Raithby and Hollands [1], the presented data are plotted in terms of $R a_{c}^{*}$ in Fig. 9. As shown in Fig. 9, Raithby and Hollands [17] correlation's under predicts the present data by $25 \%$.

## 7 Summary and conclusions

An experimental investigation of heat transfer by buoy-ancy-driven flow in an annulus bounded by concentric tubes has been conducted for different inclinations of the tubes. The annulus inner surface was maintained at high temperature while the annulus outer surface was maintained at low temperature. The investigation aims to study the effects of Rayleigh number $\left(5 \times 10^{4}<R a<5 \times 10^{5}\right)$, the annulus gap width $\left(L / D_{\mathrm{o}}=0.23-0.37\right)$ and the inclination of the annulus from the horizontal $\left(\alpha=0^{\circ}-60^{\circ}\right)$ on the heat transfer rate. The results showed that the rate of heat transfer increases with increasing the annulus gap width, decreasing the inclination of the annulus from the horizontal, and increasing the Rayleigh number. Correlations of the heat transfer enhancement due to buoyancy driven flow in an annulus has been developed in terms of the $R a, L / D_{\mathrm{o}}$ and $\alpha$. The prediction of the correlation has been compared with the present and previous data and fair agreement was found.

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